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Definition of $\tan\beta$ beyond tree-level*

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Abstract

We study the relation between different renormalization schemes for the parameter $\tan\beta$ in the Minimal Supersymmetric Standard Model. The contributions of the third-generation quark-squark loops to the differences between $\tan\beta$'s in several schemes are discussed. Their numerical differences are typically within several %.

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Some extensions of the Standard Model, including the Minimal Supersymmetric (SUSY) Standard Model[1] (MSSM), contain two Higgs boson doublets. After the spontaneous breaking of SU(2)×U(1) gauge symmetry, they get vacuum expectation values $(\langle H_1\rangle, \langle H_2\rangle) = (v_1, v_2)/\sqrt{2}$. Their ratio $\tan\beta \equiv v_2/v_1$ is a very important parameter in such models. In the MSSM, for example, masses and interactions of the SUSY particles and Higgs bosons depend[1] on $\tan\beta$, which is very crucial for theoretical predictions, experimental search for new particles and a consistency test of the theory.

In calculating radiative corrections to these observables, we have to fix a renormalization condition for $\tan\beta$. There are, however, no obvious criterion for the renormalization because $\tan\beta$ is not directly related to physical observables. Indeed, several renormalization schemes for $\tan\beta$ have been proposed in previous studies of the radiative corrections for Higgs bosons[2, 3, 4, 5, 6, 7, 8]. In this paper, we demonstrate the relations among these definitions of $\tan\beta$ in the MSSM, by showing differences between them by the one-loop contributions from the third generation quarks and squarks.

The MSSM Higgs sector lagrangian[1] has five independent parameters $(m_1^2, m_2^2, m_3^2, g_2, g_Y)$, among which m_{1-3}^2 are usually replaced with (m_A, v_1, v_2). Here m_A is the mass of the CP-odd Higgs boson A^0. We therefore need five inputs to fix these parameters. The parameters $(g_2, g_Y, \bar{v}^2 \equiv v_1^2 + v_2^2)$ are fixed by the electroweak gauge sector, as in the Standard Model. m_A is usually chosen to be the pole mass of A^0. The remaining parameter $\tan\beta \equiv v_2/v_1$ is, however, not directly related with physical observables. In order to fix it, we must either give a procedure to define the "universal", process-independent $\tan\beta$, or choose a physical input which depends on $\tan\beta$.

In the former, we can define $\tan\beta$ by renormalized $v_{1,2}$. The complexity in this procedure arises from that to obtain physical results, we have to cancel the linear terms in the Higgs potential generated by radiative correction. These linear terms from tadpole contributions can be absorbed by shifting $(v_{1,2}, m_{1-3}^2)$ but this operation is not unique[5]. For example, we can just shift $v_{1,2}$ to cancel all tadpole contributions and then apply the modified minimal subtraction. In spite of its simplicity, $\tan\beta$ in this definition runs very rapidly with the renormalization scale[9] and practically useless. Instead, we can shift both $v_{1,2}$ and m_{1-3}^2 to fulfil $\Delta v_1/v_1 = \Delta v_2/v_2$, which is so far the most popular way in studying radiative corrections in the MSSM. As for the remaining divergence corresponding to $(Z_{H_2}/Z_{H_1})^{1/2}$, we can either apply the modified minimal subtraction[2, 3, 4] (called "$\overline{\rm DR}$ scheme" here) or fix it by imposing conditions on Higgs-gauge two-point function[5, 6] ("CPR scheme").

In the latter, the on-shell definitions of $\tan\beta$ are given by one physical observable and its tree-level relation with $\tan\beta$. Here we discuss the definitions[7, 8] by leptonic partial decay widths of A^0 and of the charged Higgs boson $H^\pm$;

$$\tan\beta_{A^0} \equiv \frac{\bar{v}}{m_l}\left(\frac{8\pi}{m_A}\Gamma_{A^0\to l\bar{l}}\right)^{\frac{1}{2}}, \; \tan\beta_{H^+} \equiv \frac{\bar{v}}{m_l}\left(\frac{8\pi}{m_{H^+}}\Gamma_{H^+\to l^+\nu}\right)^{\frac{1}{2}}. \tag{1}$$

As for the renormalization of $\bar{v}$, we consider two on-shell definitions;* $1/\bar{v}^2 = \sqrt{2}G_F$ or $1/\bar{v}^2 = \bar{e}^2(m_Z^2)/(4m_W^2(1-m_W^2/m_Z^2))$. Each definition in Eq. 1 has then two versions. The another on-shell definition of $\tan\beta$ by the sfermion mass splitting[2, 3] does not work[2, 11] for large $\tan\beta$ and not discussed here.

Here we consider the contributions of quarks and squarks in the third generation to the differences between $\tan\beta$'s given above. The counterterm for $\tan\beta \equiv t_\beta$ in each scheme then takes the following form[2, 3, 4, 5, 7, 8, 12];

$$\begin{aligned}
\delta(t_\beta)/t_\beta|_{\overline{\rm DR}} &= -3(16\pi^2)^{-1}(m_t^2/v_2^2 - m_b^2/v_1^2)(2/(4-d) - \gamma_E + \ln 4\pi), \\
\delta(t_\beta)/t_\beta|_{\rm CPR} &= (2m_Z)^{-1}(t_\beta + t_\beta^{-1}){\rm Re}\Pi_{ZA}(m_A^2), \\
\delta(t_\beta)/t_\beta|_{A^0} &= {\rm Re}[m_Z^{-1}t_\beta^{-1}\Pi_{ZA}(m_A^2) + \Pi'_A(m_A^2)/2] + \delta\bar{v}/\bar{v}, \\
\delta(t_\beta)/t_\beta|_{H^+} &= {\rm Re}[-m_W^{-1}t_\beta^{-1}\Pi_{W^-H^+}(m_{H^+}^2) + \Pi'_{H^\pm}(m_{H^+}^2)/2] + \delta\bar{v}/\bar{v}. \qquad (2)
\end{aligned}$$

$\Pi_{\dots}(q^2)$ are unrenormalized two-point functions of Higgs and gauge bosons.

Figure 1 shows numerical values of various $\tan\beta$ given in Eq. 2 as functions of $\tan\beta_{\overline{\rm DR}}(m_Z)$. As we included only the $(t, b, \tilde{t}, \tilde{b})$ contributions, the relative differences between $\tan\beta$'s are large for small $\tan\beta$ (large top-Higgs Yukawa coupling). We can see that the differences are within several %, the same order as the running of $\tan\beta_{\overline{\rm DR}}$ from m_Z to m_A. Main parts of the differences come from the quark loops. As for the definitions by the leptonic decay widths, differences by the definition of $\bar{v}$ are quite significant.

We have discussed the relation between $\tan\beta$'s in different renormalization schemes in the MSSM. We have shown that their numerical differences by the $(t, b, \tilde{t}, \tilde{b})$ loops are typically within several % for commonly used schemes. More complete study will be presented elsewhere[12].

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*The effective coupling[10] $\bar{e}^2(m_Z^2)$ is used here to absorb large QED correction.

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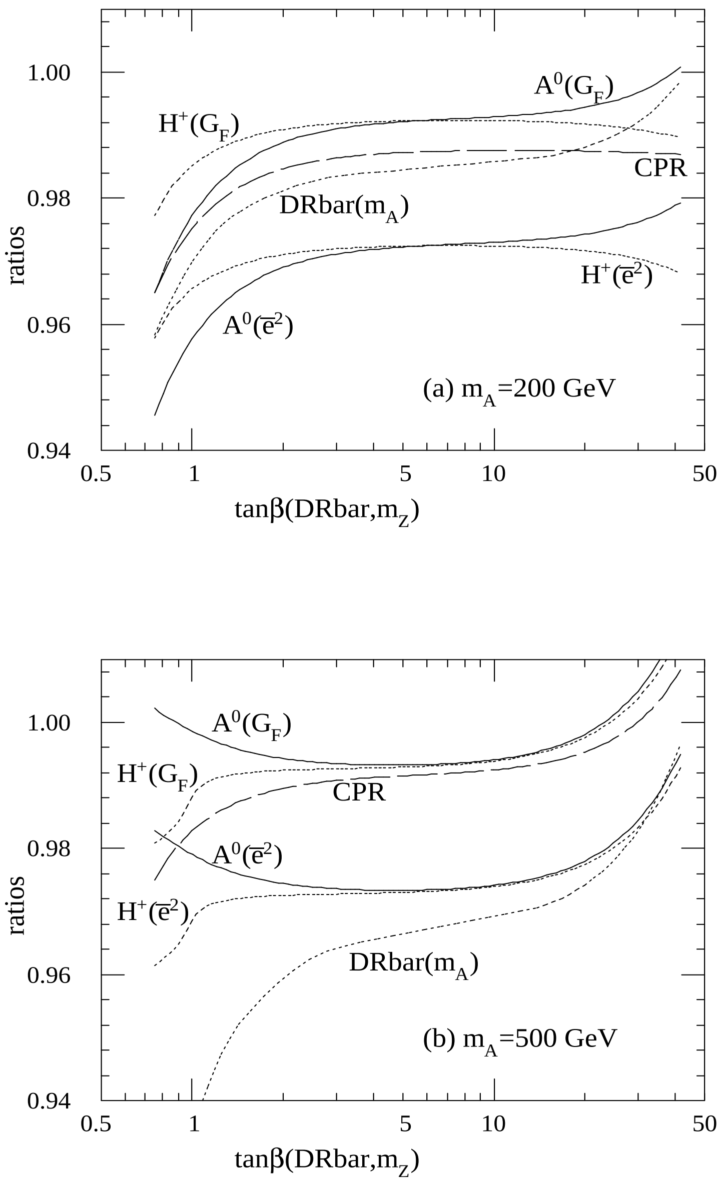

Figure 1: Ratios of $\tan\beta$ in several definitions to $\tan\beta_{\overline{\rm DR}}(m_Z)$ for $(M_{(\tilde{Q},\tilde{U},\tilde{D})},\ \mu,\ A_{(t,b)})$(GeV)= (300, 400, 0) and m_A(GeV) =200 (a), 500 (b).